1

Granger causality and GARCH models

Ewa M. Syczewska[2] and Zbigniew R. Struzik[345]

8th FENS Sympozjum, November 6, 2015

[2]Warsaw School of Economics, Department of Economic Analyses, Institute of Econometrics, Madalinskiego 6/8, 02-513 Warsaw, Poland; ewams@sgh.waw.pl

[3]RIKEN Brain Science Institute, 2-1 Hirosawa, Wako-shi 351-0198, Japan

[4]Graduate School of Education, The University of Tokyo, 7-3-1 Hongo, Bunkyo-ku, Tokyo 113-0033, Japan

[5]Institute of Theoretical Physics and Astrophysics, The University of Gdańsk, Wita Stwosza 57, 80-952 Gdansk, Poland

[1]Presented at 8 Sympozjum FENS 2015, November 4-7, 2015, Rzeszów

Approaches to Wiener-Granger causality

Let us remind that there are several approaches to causality:

- the autoregressive and/or spectral based Granger method [Granger CWJ, Econometrica 37(3)]
- the information-theoretic oriented Transfer Entropy approach by Schreiber (2000)], generalized by Barnett et al., Phys. Rev. Lett. 103(23), and by Hlavackova-Schindler et al. (2007, 2011);
- Judea Pearl approach based on graph theory;
- Hoover approach based on multivariate macroeconometric models, etc.

Example: results for EURPLN, WIG20 and DAX

Source of daily data: http://stooq.pl

- To test G-causality between bilateral exchange rate and corresponding stock indices, we take into account nonstationarity of the series, hence we test causality for the returns:

$$r_t = \log P_t - \log P_{t-1} \quad (1)$$

  where P_T – price of intrument at time t.

- Financial data are influenced by the crisis. Olbryś and Majewska [2014] in the research based on turning points of the main stock indices, show that the common crisis period for the U.S. and the main European economies is October 2007-February 2009[18].

[18] Olbryś, J., E. Majewska (2014) Direct identification of crisis period on the CEE stock markets: The Influence of the 2007 U.S. subprime crisis, Procedia Economics and Finance, Special Issue: International Conference on Applied Economics, ICOAE2014, Vol. 14, 461470.

Volatility of financial series

We use daily data, hence the empirical distribution is not Gaussian (skewed, leptokurtic, with heavy tails).
The returns r_t show the ARCH effect, known as variance clustering: changing conditional variance, with autoregressive behavior.

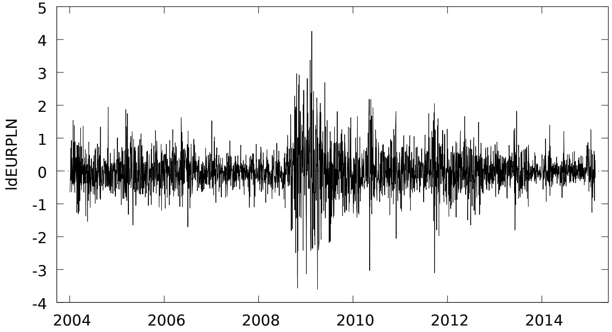

Figure : Log returns of the EURPLN exchange rate.

GARCH prefiltering

In our earlier research (Syczewska annd Struzik [APPA, 2015]) causal relationship between financial variables has been shown for returns of bilateral exchange rates and stock indices. In financial econometric literature (Kliber (2011), Fiszeder and Orzeszko (2012) among others), prefiltering the series with ARMA-GARCH models is first applied. Doman and Doman (2013) apply GJR-GARCH model to stock indices returns, to check their possible interdependence.

USDPLN and the Polish and U.S. index

The Granger causality test results [Syczewska2014] show that during crisis, Oct. 01,2007- Feb.27 2009, the null hypothesis of no causality from WIG20 to USDPLN is weakly rejected, the same for causality from USDPLN towards WIG20. After the crisis, for period March 02 2009–July 26 2010, H_0 of no causality from WIG20 towards USDPLN exchange rate is not rejected , and lack of causality from USDPLN towards WIG20 is strongly rejected. In all cases, normality both of the variables and error terms is strongly rejected. Here we want to check if similar pattern holds for EURPLN and the corresponding Polish and European stock indices. In addition, we test causality for volatilities.

Linear Granger causality test

[Barnett, Barrett, 2009] and [Hlávačková-Schindler 2011]: the Granger causality is described in the framework of linear regression and VAR (vector-autoregressive) models.
Let Y – discrete time-series, the effect variable, and X – "cause" variable. To test G-causality, run a regression of Y on X and/or its lags, and check if (lags of) X are significant (if all their parameters are different from zero). Usually the joint significance test is applied, with χ^2 or Fisher F distribution:

$$y_t = \sum_{j=1}^{k} \alpha_j y_{t-j} + \sum_{j=1}^{k} \beta_j x_{t-j} + \varepsilon_t \tag{2}$$

where: y_t – observations of a variable of interest; x_{t-j} – observations of causal variable; α_j, β_j – parameters of the regression; ε_t – error term. The null hypothesis $H_0 : \beta_j = 0$ for all $j = 1, ..., k$ corresponds to lack of G-causality from X to Y, the alternative $H_1 : \beta_j \neq 0$ for some $j \in \{1, ..., k\}$ means that the X Granger-causes Y.

Testing G-causality in VAR framework – 1

Let **Y** denote column vector of variables. The VAR (Vector Autoregressive) model equations use lags of all variables up to k-th as explanatory variables:
$\mathbf{Y} = [Y_1, Y_2, \ldots, Y_m]^T$.

$$\mathbf{Y}_t = \mathbf{A}_0 + \mathbf{A}_1\mathbf{Y}_{t-1} + ... + \mathbf{A}_k\mathbf{Y}_{t-k} + \varepsilon_t \qquad (3)$$

where **Y** – vector of variables of the model, $\mathbf{A}_i$ – matrices of parameters, k – number of lags, ε – (column) vector of error terms, $\varepsilon_t = [\varepsilon_{1t}, \varepsilon_{2t}, \ldots, \varepsilon_{mt}]$. The test of non-causality in the context of bivariate VAR ($\mathbf{Y} = [Y_1, Y_2]^T$) is the Wald test of joint insignificance of all lags of Y_2 in the equation for Y_1.

Nonlinear Granger causality

One of nonlinear causality tests is the Diks-Panchenko[2006] method:

- lack of causality means that the distribution of Y_{t+1} conditional on Y_t, X_t is the same as conditional distribution on Y_t alone;
- propose approximation based on index function.

The null of no causality is rejected if the test statistic is high.

Our results for EURPLN, DAX and WIG20 – (1)

Linear causality test for log returns, daily data since October 2007 until February 2009.

Variable (X,Y)	Test statistic	Causality?
DAX EURPLN:	F(5, 340) = 1,5108 [0,1858]	no
EURPLN DAX:	F(5, 340) = 1,2104 [0,3038]	no
WIG20 EURPLN:	F(5, 340) = 2,1712 [0,0569]	yes
EURPLN WIG20:	F(5, 340) = 3,4269 [0,0049]	yes
DAX WIG20:	F(5, 340) = 1,5178 [0,1836]	no
WIG20 DAX:	F(5, 340) = 1,1353 [0,3413]	no

Table : Results of linear causality test for the crisis period

Source of data: stooq.pl

Our results for EURPLN, DAX and WIG20 – (2)

Linear causality tests for **volatility** (computed as conditional variance, based on the GARCH(1,1) or GARCH(1,2) models for the returns).

Variable (X,Y)	Test statistic [p-value]	Causality?
hldDAX hldEURPLN:	F(5, 335) = 1.6489 [0.147]	no
hldEURPLN hldDAX:	F(5, 335) = 2.4566 [0.033]	yes
hldWIG20 hldEURPLN:	F(5, 335) = 1.8523 [0.102]	no
hldEURPLN hldWIG20:	F(5, 335) = 7.5502 [0.000]	yes
hldDAX hldWIG20:	F(5, 335) = 4.1144 [0.0012]	yes
hldWIG20 hldDAX:	F(5, 335) = 2.0144 [0.076]	yes

Table : Results of linear causality test for volatilities

Source of data: stooq.pl

Our results for EURPLN, DAX and WIG20 – (3)

Next we apply **the nonlinear Diks-Panchenko test to the volatilities**:
Null hypothesis: volatility of the X does not cause volatility of the Y variable.
Low p-value indicates causality.

Variable (X,Y)	Test statistic [p-value]	Causality?
hldDAX hldEURPLN	T statistics=0.833 [0.202]	no
hldEURPLN hldDAX	T statistics=2.550 [0.0054]	yes
hldWIG20 hldEURPLN	T statistics=1.160 [0.123]	no
hldEURPLN hldWIG20	T statistics=2.105 [0.018]	yes
hldDAX hldWIG20	T statistics=2.052 [0.0201]	yes
hldWIG20 hldDAX	T statistics=3.764 [0.00008]	yes

Table : Results of nonlinear causality test for volatilities

Concluding remarks

- L. Barnett, T. Bossomaier, Phys. Rev. Lett. 109, 138105 (2012) mention the GARCH properties of time series, sayng that “it is not clear” how the Granger causality test should be applied in such case.
- On the other hand, there is huge amount of financial econometric literature on this subject: the GARCH filtering is applied, thus leading to Granger-causality testing also for volatilities (see, e.g., Osińska book on causality tests; Doman M. and Doman R., or Fiszeder P., or Orzeszko W. books and papers on this subject). D. Serwa uses similar tools for detecting interdependencies between risks for markets.
- We give here the results of Granger-causality test for financial returns and for volatilities of stock indices and the bilateral exchange rate. The linear test and the nonlinear Diks-Panchenko approach in most cases give the same results.
  The results can be extended to the GARCH models with several non-Gaussian distributions.

Thank you!