Lifetime of Correlations between Stocks

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Abstract

The correlation coefficient between stocks depends on price history and includes information on hierarchical structure in financial markets. It is useful for portfolio selection and estimation of risk. I introduce the Life Time of Correlation between stocks prices to know how far we should investigate the price history to obtain the optimal durability of correlation. I carry out my research on emerging (Poland) and established markets (in the USA and Germany).

The correlation coefficient

The Correlation coefficient defines degree of similarity between the synchronous time evolution of a pair of stocks prices:

$$\rho_{ij} = \frac{<Y_iY_j> - <Y_i><Y_j>}{\sqrt{\left(<Y_i^2> - <Y_i>^2\right)\left(<Y_j^2> - <Y_j>^2\right)}}$$

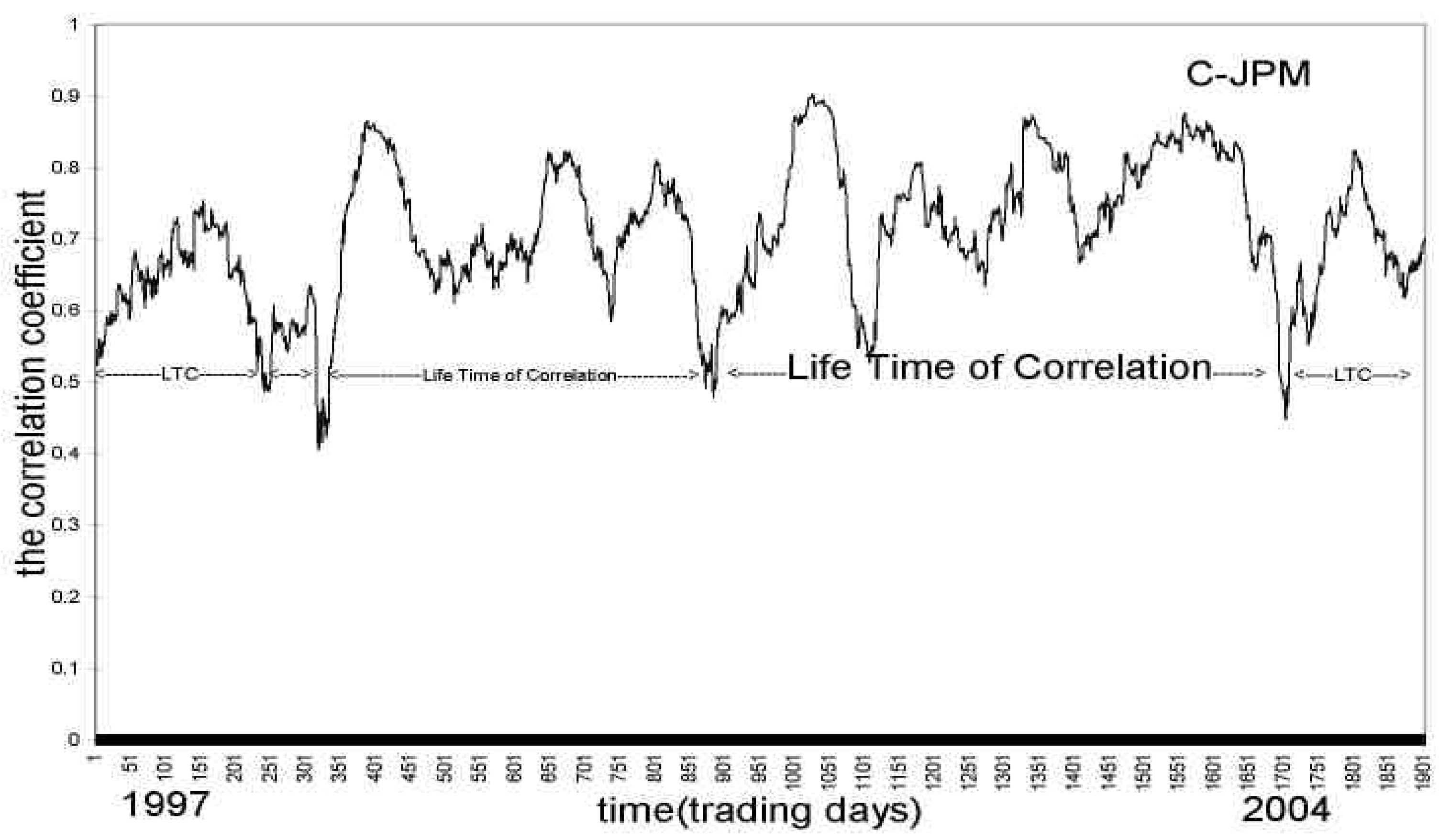
where i and j are the numerical labels of stocks, Y_{ij} is the price return. $\rho(t,\Delta t)$ also depends on the window width Δt.

The lifetime of correlation between stocks

The Lifetime $\tau_{ij}(\Delta t)$ of Correlation (LTC) is defined as a length of time when the correlation coefficient $\rho(t,\Delta t)$ is permanently on the strong level

$$\rho \in \left\langle \frac{1}{2}, 1 \right\rangle$$

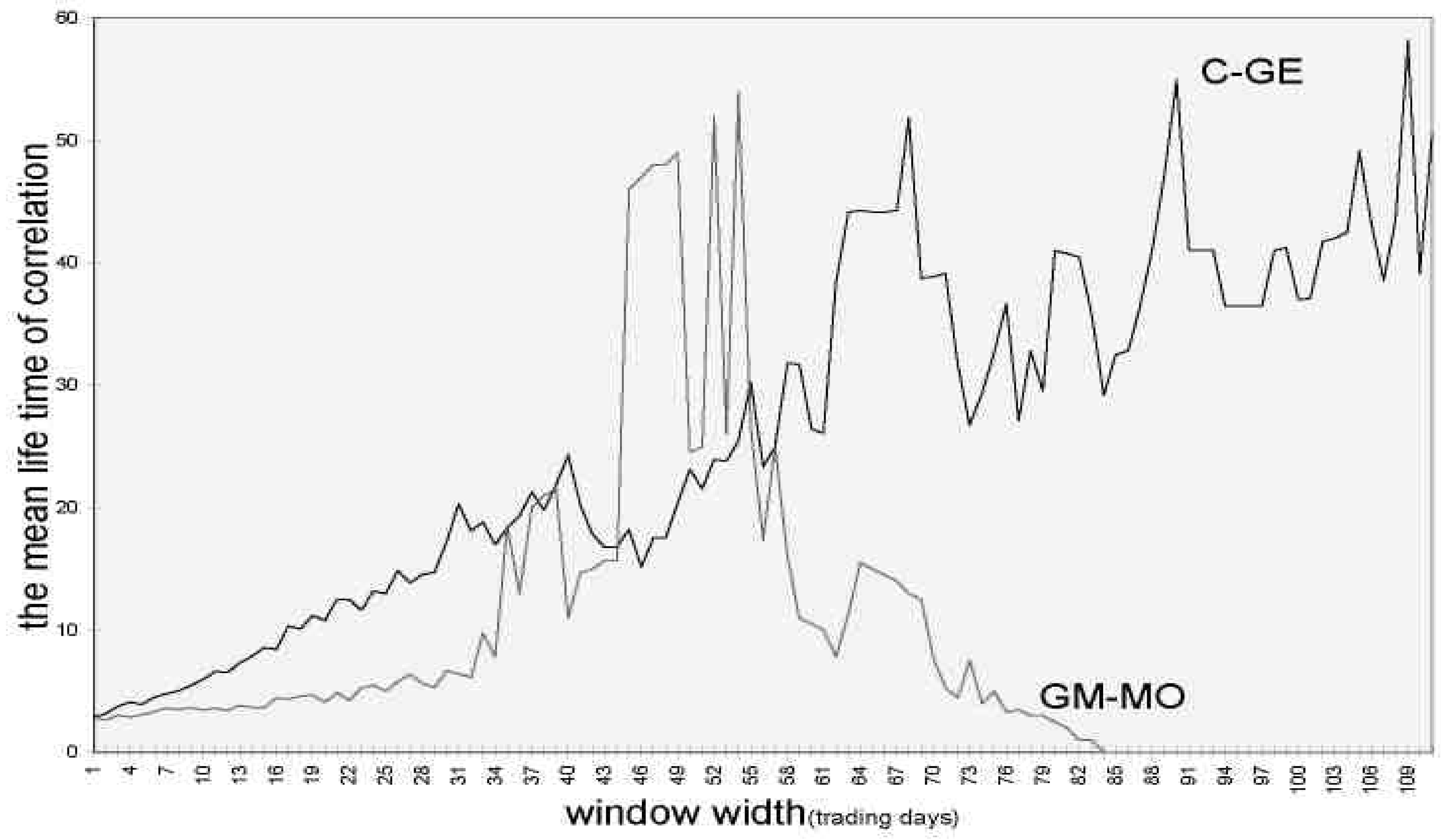
For instance, when Δt = 65 the correlation coefficient $\rho(t)$ between C-JPM from DJIA portfolio reaches the strong level six times:



For each pair it is possible to introduce the mean lifetime of correlation (MLTC):

$$\langle \tau_{ij}(\Delta t) \rangle = \frac{1}{N} \sum_{k=1}^{N} \tau_{ijk}(\Delta t)$$

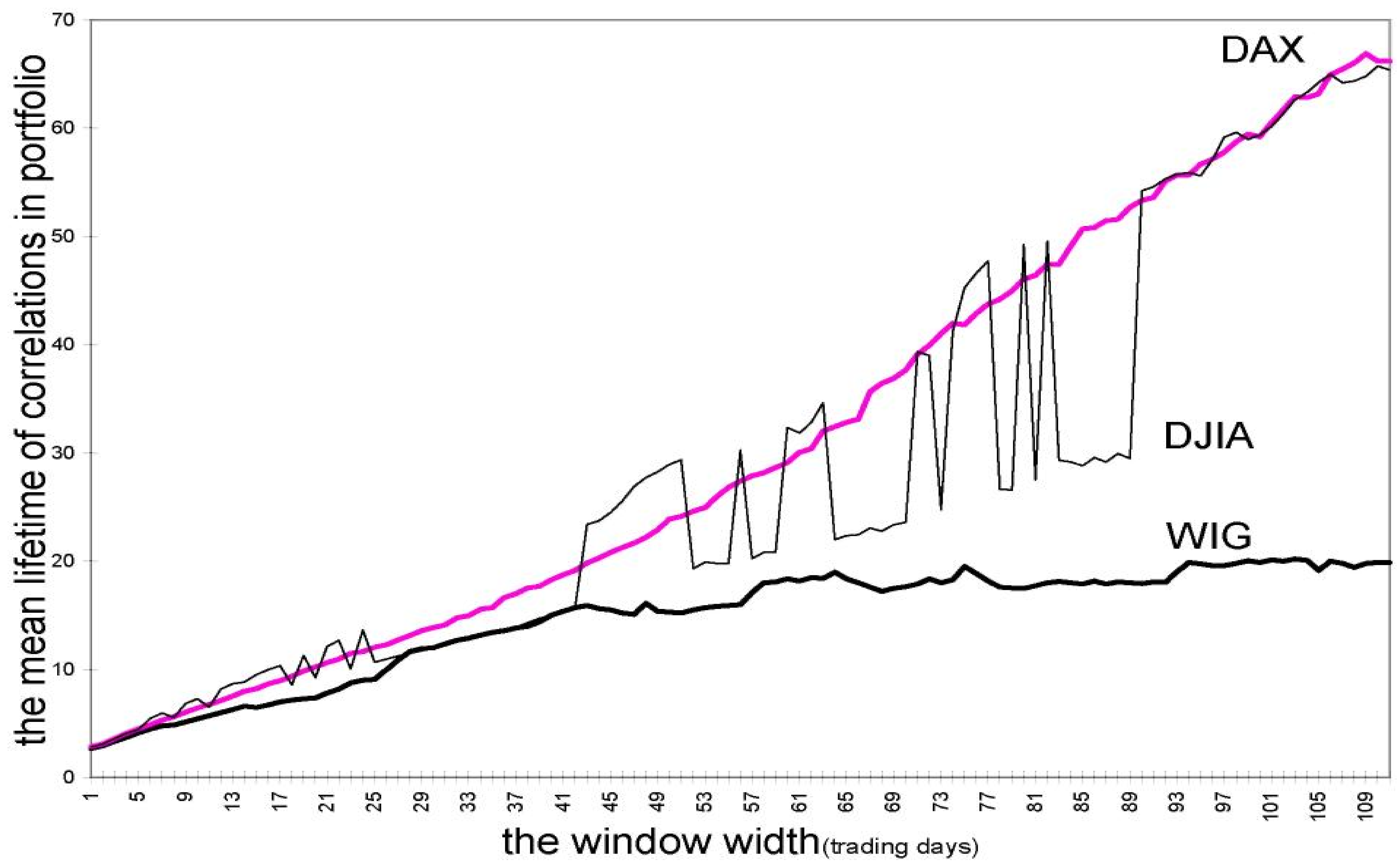
where k enumerates the successive lifetimes of correlation. i,j are the numerical labels of stocks. Like $\rho_{ij}(t,\Delta t)$, the mean lifetime of correlation depends on the window width Δt. For instance, when Δt is equal 65, the mean life time of correlation $\tau_{C\text{-}JPM}$ = 233,125 trading days.



For strongly correlated pairs, like C-GE from the DJIA, the mean lifetime of correlation grows up with Δt. On the other hand, weakly correlated pairs like GM-MO have their lifetime equal to zero for sufficiently large Δt > 65 trading days. It is also visible that when Δt is too small, the mean lifetimes for all two pairs are small and very similar because of a very short price history and price fluctuations. So, it is hard to distinguish their levels of correlations. For too large Δt, however, the MLTC has too large variations. Thus, small increase of Δt causes very dramatic change of $\tau_{ij}(t,\Delta t)$. This phenomenon has been discovered for all investigated pairs of stocks traded in all three markets (DJIA, DAX, WIG). This observation suggests that the window width shouldn't be too large.

The mean lifetime of correlations in portfolios

It is also possible to introduce the mean lifetime of correlations inside set of stocks used to compute main indices DJIA, DAX, WIG



From a practical point of view, it is more useful to analyse the standard deviation from MLTC in each portfolio. It allows to find an optimal window width Δt to detect the variety of correlations. Thus, it is reasonable to choose at least 3 or 4 months from the latest data series to compute correlation coefficients efficiently.