Characteristics of fluctuations for the stock and currency markets

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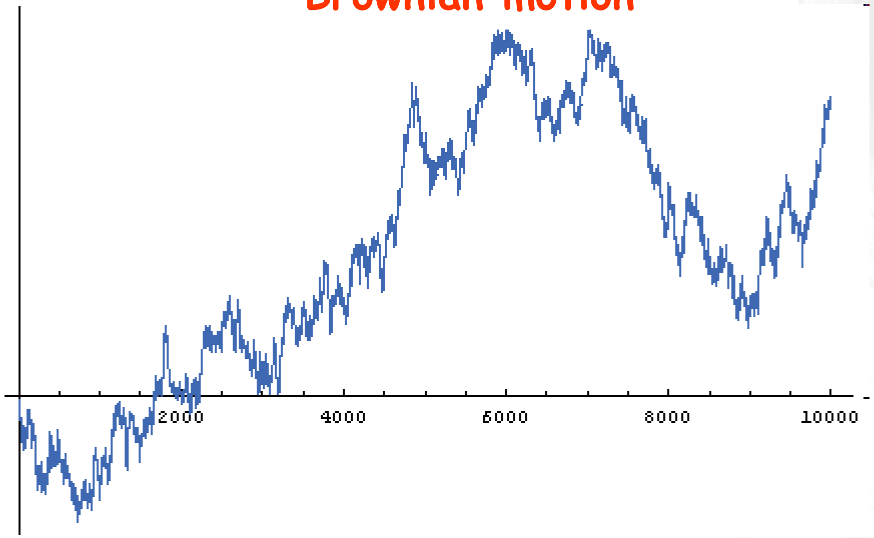
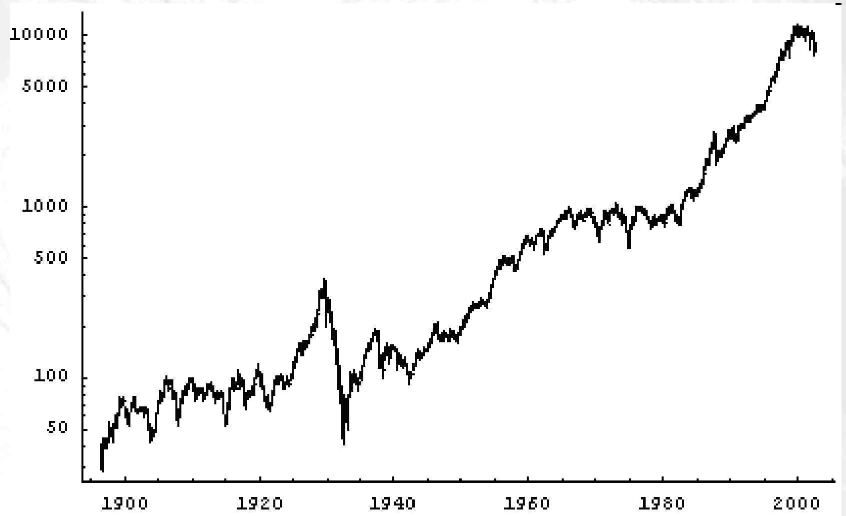
Fluctuations

Brownian motion

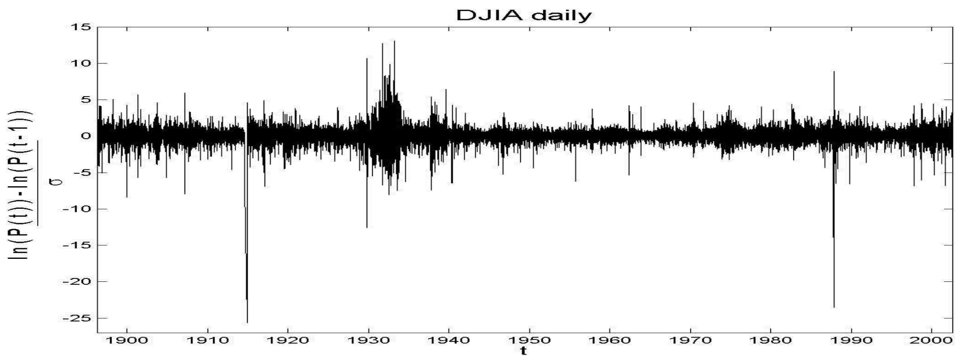


Dow Jones



Brownian motion – Gaussian fluctuations

Returns – financial fluctuations



Data

First part

Returns and trading volume of 8 largest Polish stock market companies over the period 2000.11.17 - 2008.03.06 for the time lags ranging from 1 minute and 2 hours.
We study this for tick-by-tick time series too.

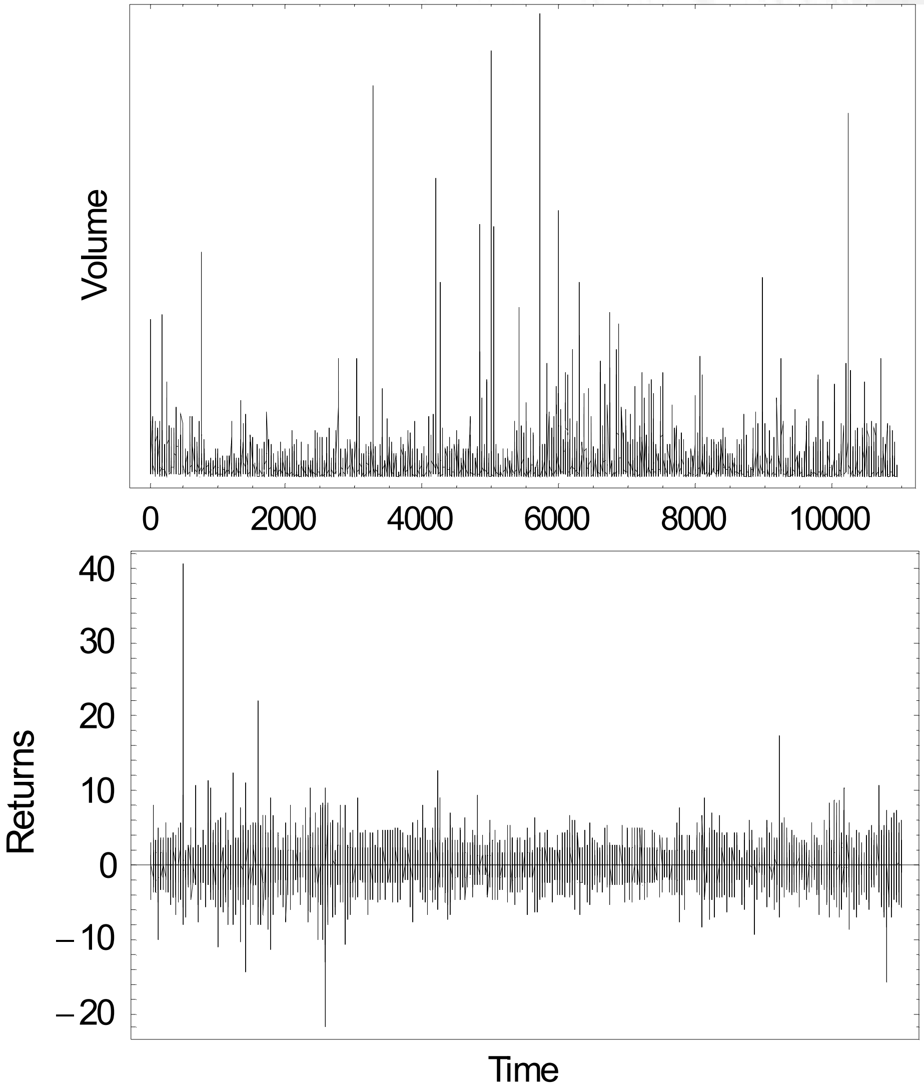
Second part

High frequency returns from the foreign exchange (FOREX) market. This study is based on three exchange rates:

CHF/JPY, EUR/GBP, GBP/USD

The FX data used in the present analysis are sampled with 1 minute frequency over the period from 2004.01.01- 2009.06.30. It’s almost 3000000 records for each exchange rate.

Returns and trading volume of Polish companies



Does any universal relationship

between

large fluctuations

of returns and trading volume

exist

Theory

X. Gabaix, P. Gopikrishnan, V. Plerou, H.E. Stanley, ***„A theory of power law distributions in financial market fluctuations”***, *Nature 423,* (*2003*), 267–270.

X. Gabaix, P. Gopikrishnan, V. Plerou and H. E. Stanley, **„Institutional Investors And Stock Market Volatility”,** Quarterly Journal of Economics, (2006), v121, 461-504.

r - return

V - volume (the number of shares traded)

For large volumes ***V*** they hypothesize functional form:

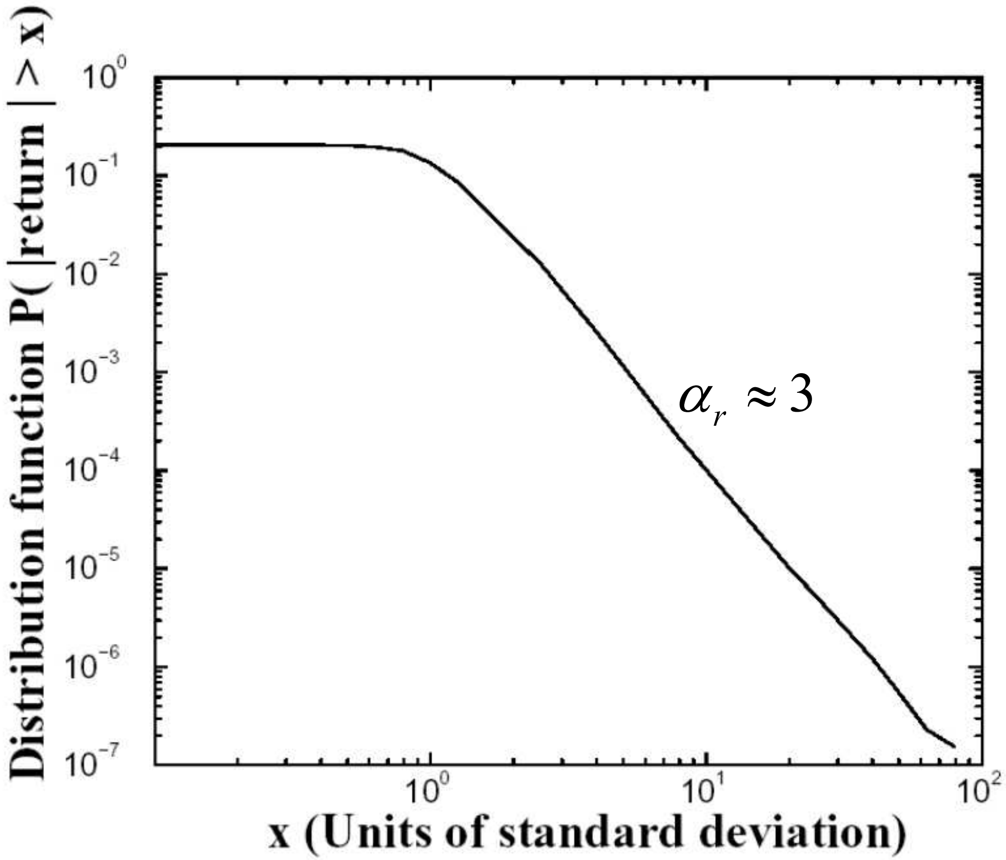
$$r \approx kV^{1/2}$$ where k is some constant

$$P\left(|r| > x\right) \sim x^{-\alpha_r}$$

$$P\left(kV^{1/2} > x\right) = P\left(V > \left(xk^{-1}\right)^2\right) \sim x^{-2\alpha_V}$$

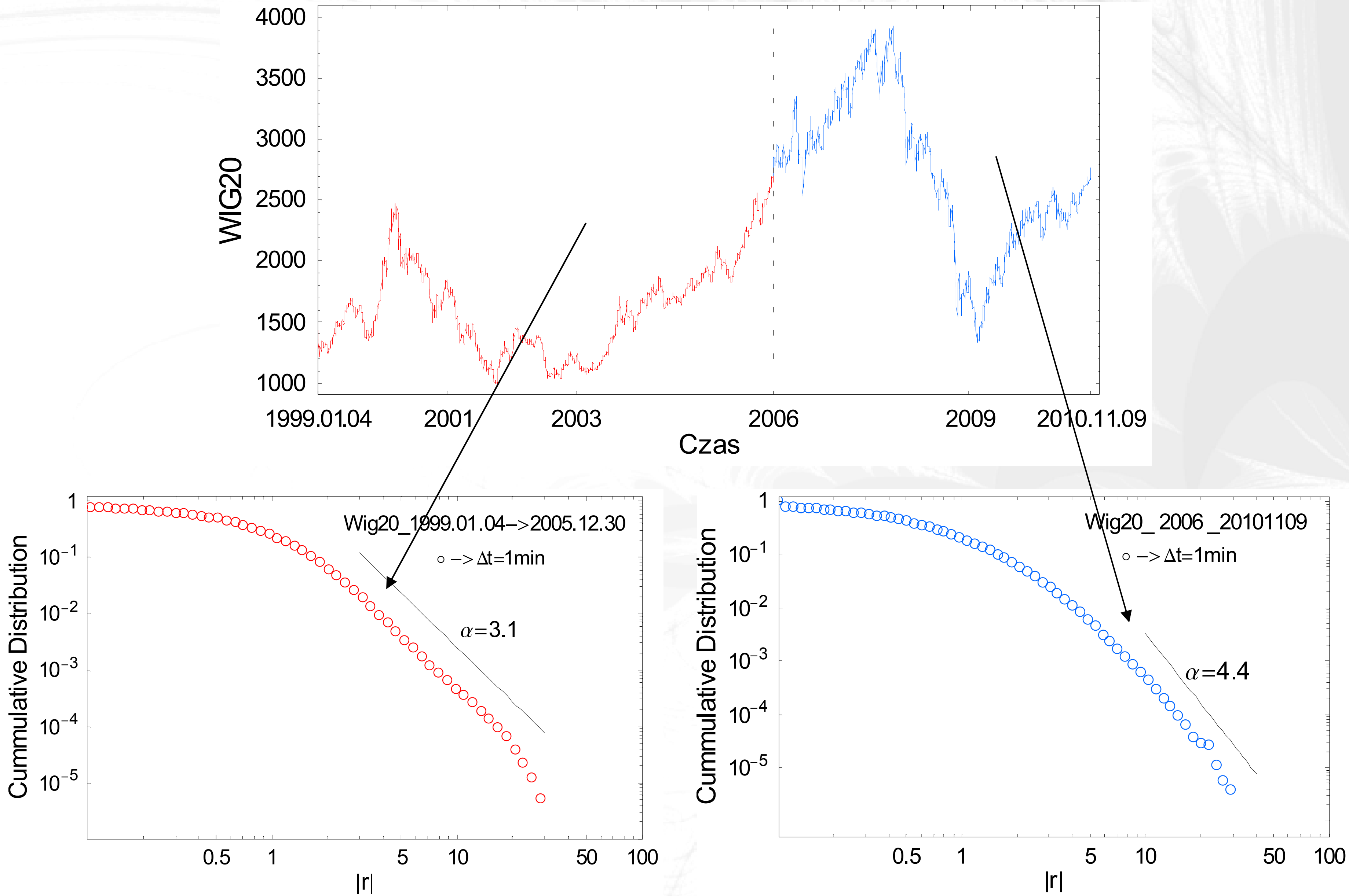
$$\frac{\alpha_r}{\alpha_V} \approx 2$$

Empirical cummulative distribution



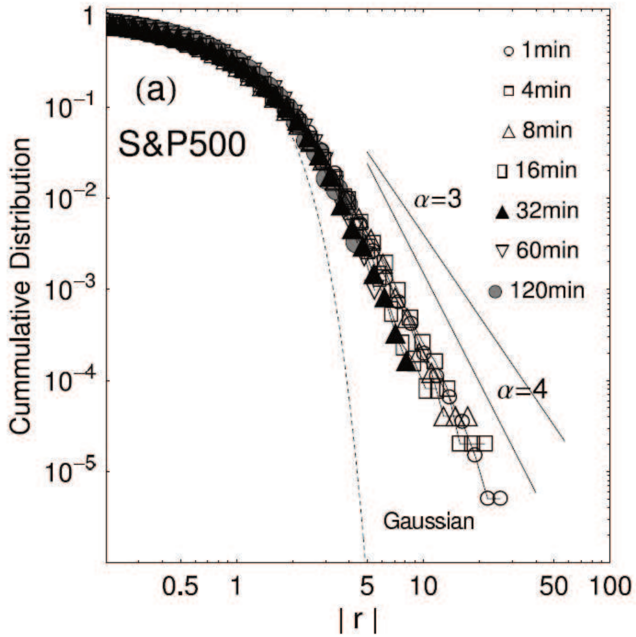
$$\frac{\alpha_r}{\alpha_V} \approx 2$$

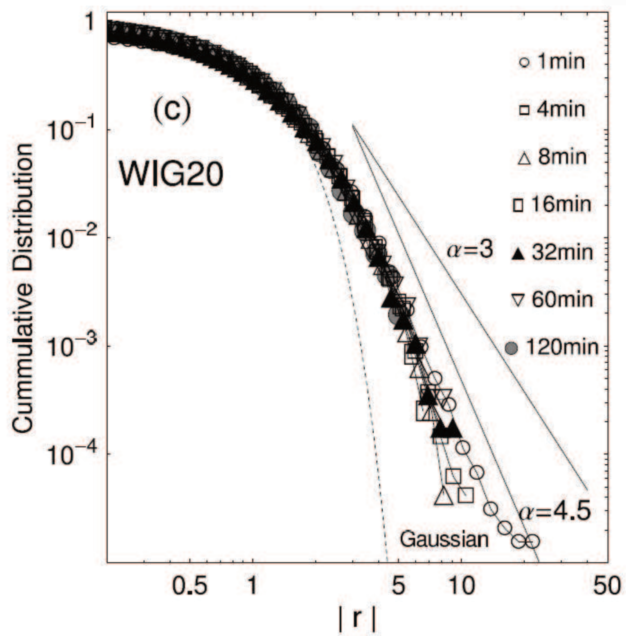
Figure I: Empirical cumulative distribution of the absolute values of the normalized 15 minute returns of the 1,000 largest companies in the Trades And Quotes database for the 2-year period 1994–1995 (12 million observations). We normalize the returns of each stock so that the normalized returns have a mean of 0 and a standard deviation of 1. For instance, for a stock i, we consider the returns $r'_{it} = (r_{it} - r_i)/\sigma_{r,i}$, where r_i is the mean of the r_{it}'s and $\sigma_{r,i}$ is their standard deviation. In the region $2 \leq x \leq 80$ we find an ordinary least squares fit $\ln P(|r| > x) = -\zeta_r \ln x + b$, with $\zeta_r = 3.1 \pm 0.1$. This means that returns are distributed with a power law $P(|r| > x) \sim x^{-\zeta_r}$ for large x between 2 and 80 standard deviations of returns. Source: Gabaix et al. [2003].



If the relationship is still true, if the inverse cubic power law does not observe ?

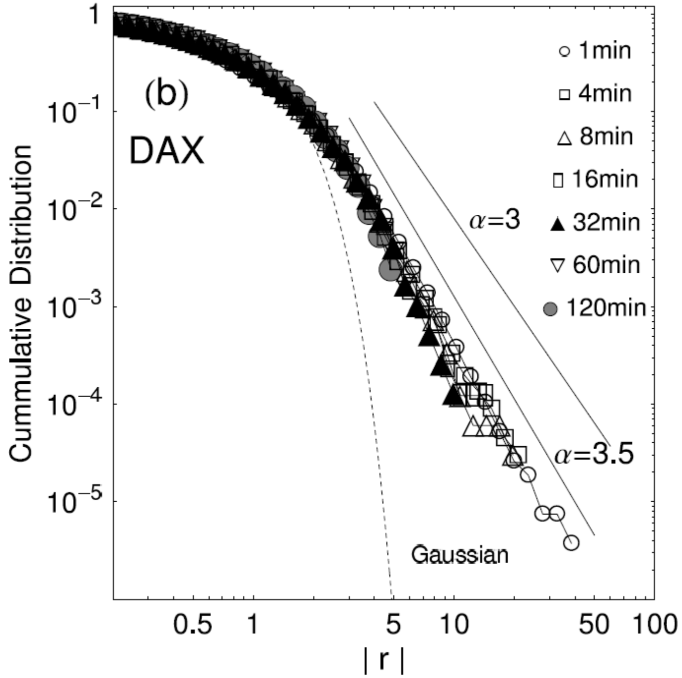
$$\frac{\alpha_r}{\alpha_V} \approx 2 \;?$$





S. Drozdz, M. Forczek, J. Kwapien, P. Oswiecimka, R. Rak,
Stock market return distributions: from past to present
Physica A 383, 59-64 (2007)

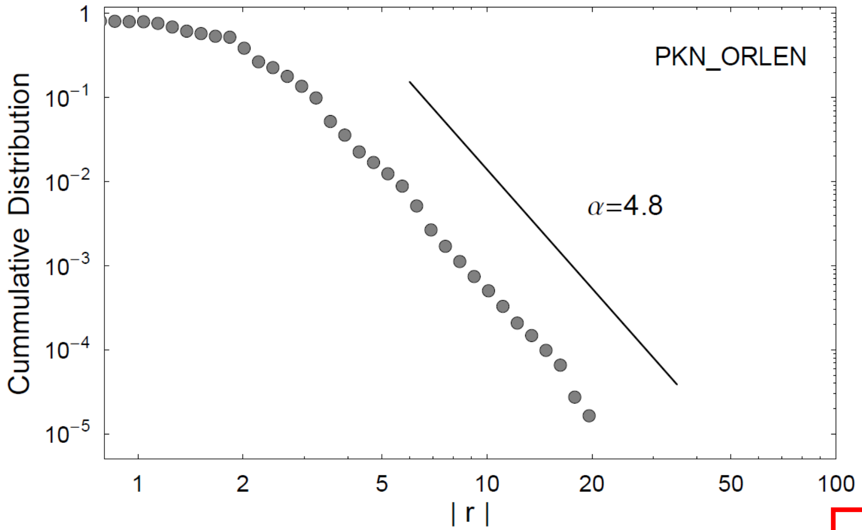
S&P500, DAX and WIG20 indices over the interval May 2004 - May 2006

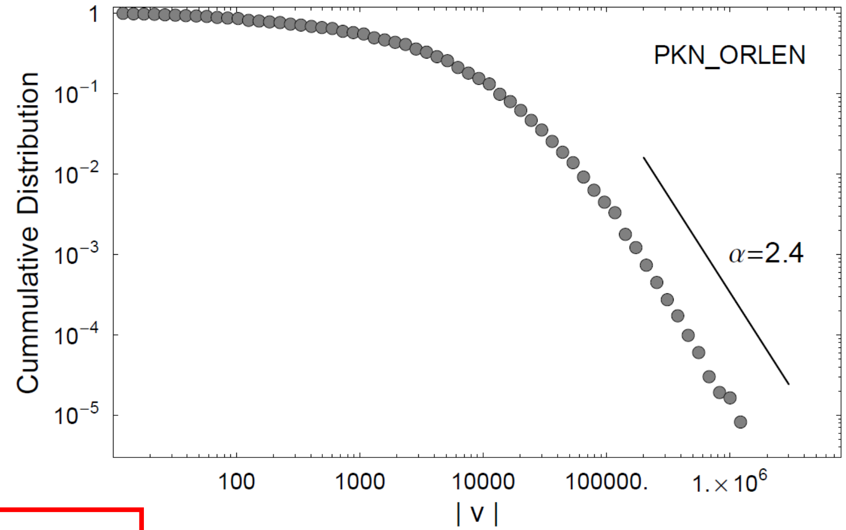


If the relationship is still true, if the inverse cubic power law does not observe ?

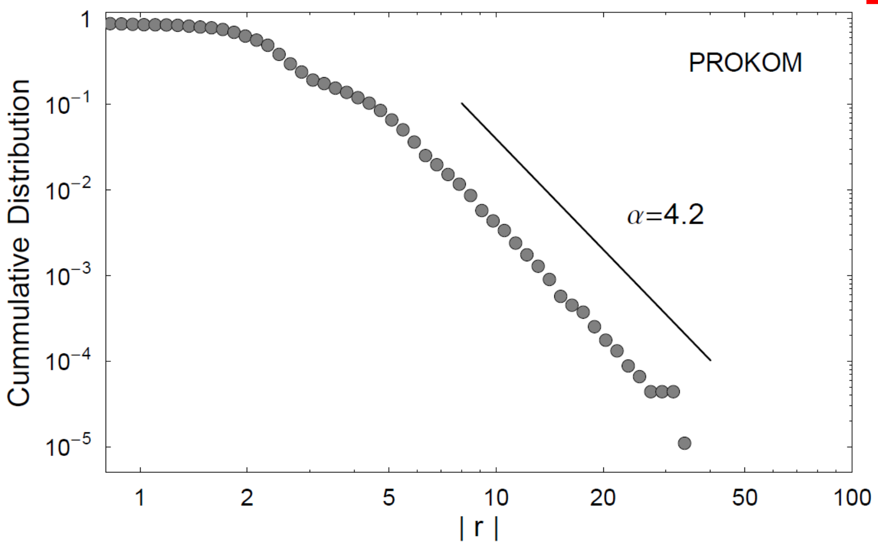
$$\frac{\alpha_r}{\alpha_V} \approx 2$$

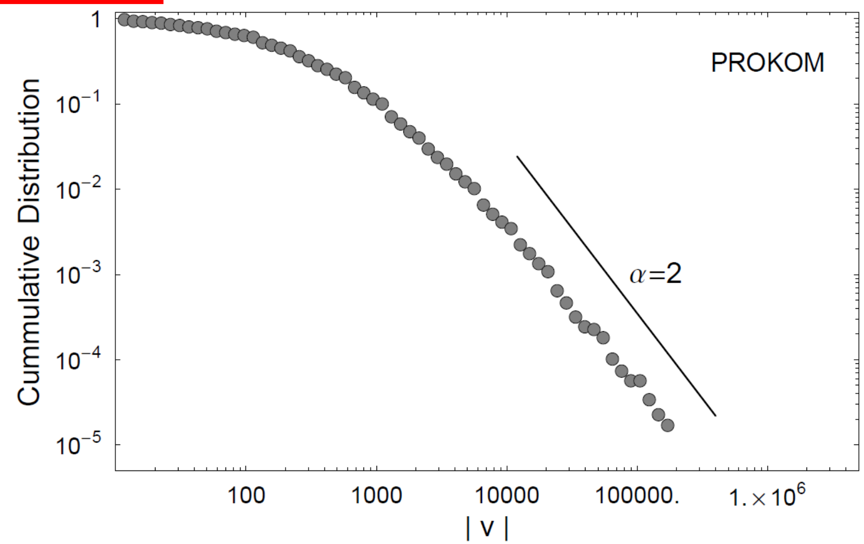
Cummulative distributions of the returns and volumes of the Polish stock market companies over the period 2000.11.17 - 2008.03.06 for 1 minute time lag

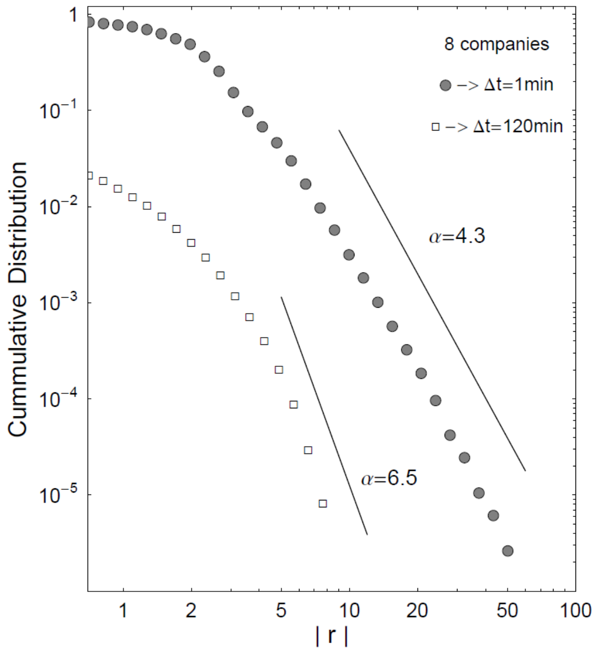


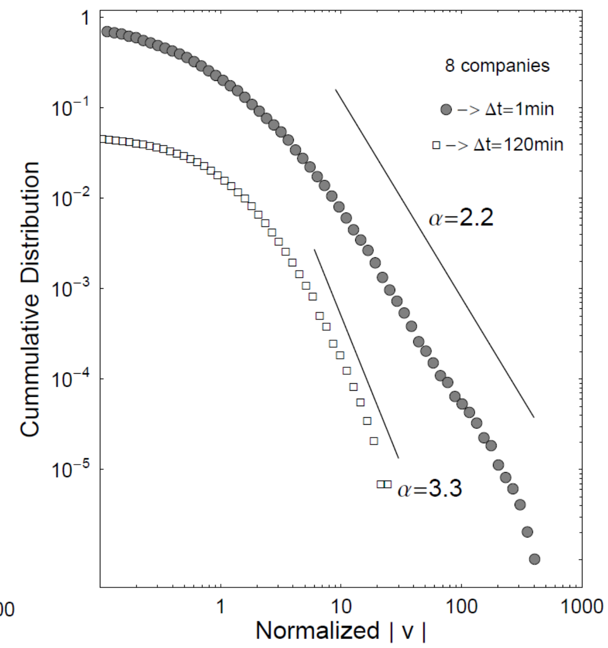


$$\frac{\alpha_r}{\alpha_V} \approx 2$$

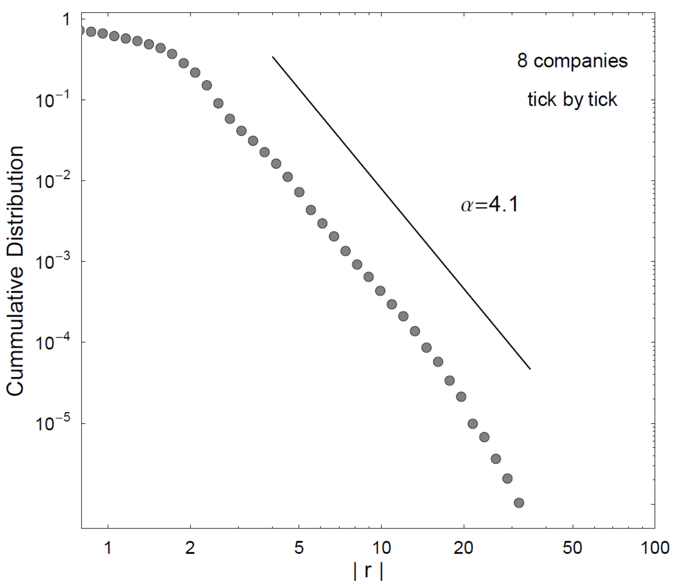


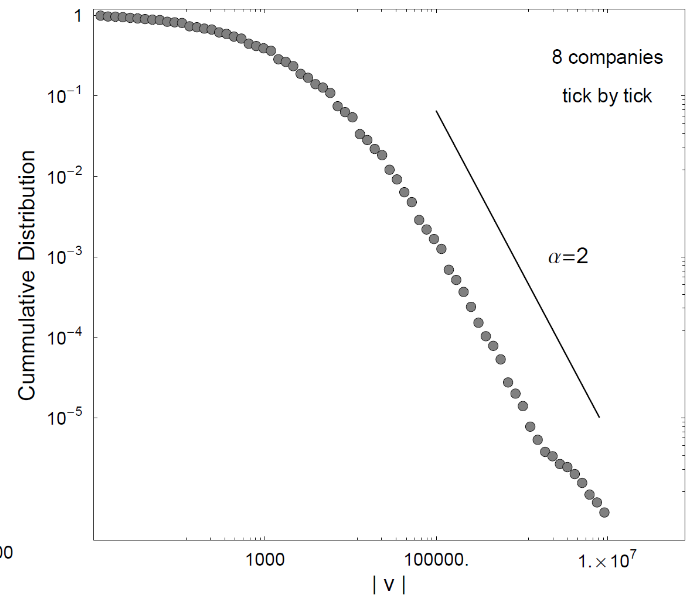






$$\frac{\alpha_r}{\alpha_V} \approx 2$$





Nonextensive Entropy & q-Gaussians

$$S_q = -\sum_{i=1}^{N} p_i^q \ln_q p_i$$

C. Tsallis (1988)

$$\ln_q x = (x^{1-q} - 1)/(1-q)$$

$$S_{q\to 1} = S_{BG}$$

$$p(x) = \mathcal{N}_q \, e_q^{-\mathcal{B}_q (x-\bar{\mu}_q)^2}$$

$$e_q^x = [1 + (1-q)\, x]^{\frac{1}{1-q}}$$

$$p_{q\to 1}(x) \to Gaussian$$

$$p(x) \sim x^{\frac{2}{1-q}}$$

For $q = 3/2$ we obtain inverse cubic power low

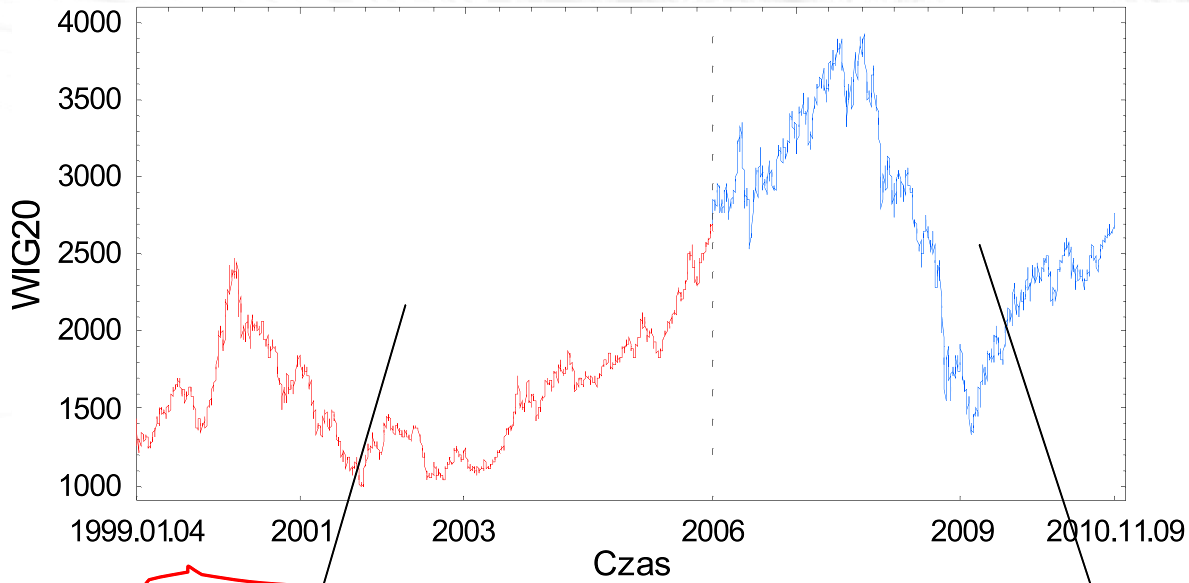
$$P_\pm(x) = \mp \int_{\pm\infty}^{x} p(x')dx' \longrightarrow$$

$$P_\pm(x) = \mathcal{N}_q \left(\frac{\sqrt{\pi}\, \Gamma\left(\frac{1}{2}(3-q)\ \beta\right)}{2\ \Gamma(\beta)\ \sqrt{\frac{\mathcal{B}_q}{\beta}}} \pm (x - \bar{\mu}_q)\ {}_2F_1(\alpha, \beta; \gamma; \delta) \right)$$

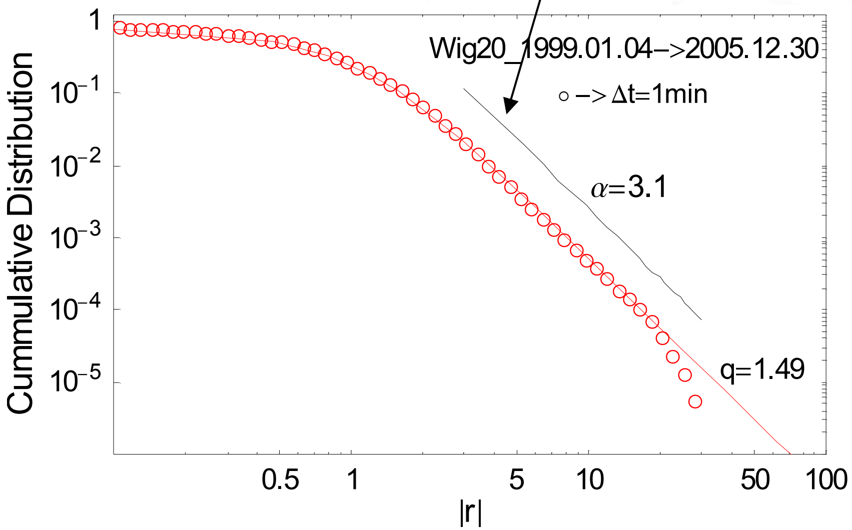
$${}_2F_1(\alpha,\beta;\gamma;\delta) = 1 + \frac{\alpha\beta}{1!\,\gamma}\delta + \frac{\alpha(\alpha+1)\beta(\beta+1)}{2!\,\gamma(\gamma+1)}\delta^2 + \ \dots \ = \sum_{k=0}^{+\infty} \frac{\delta^k (\alpha)_k (\beta)_k}{k!\,(\gamma)_k}$$

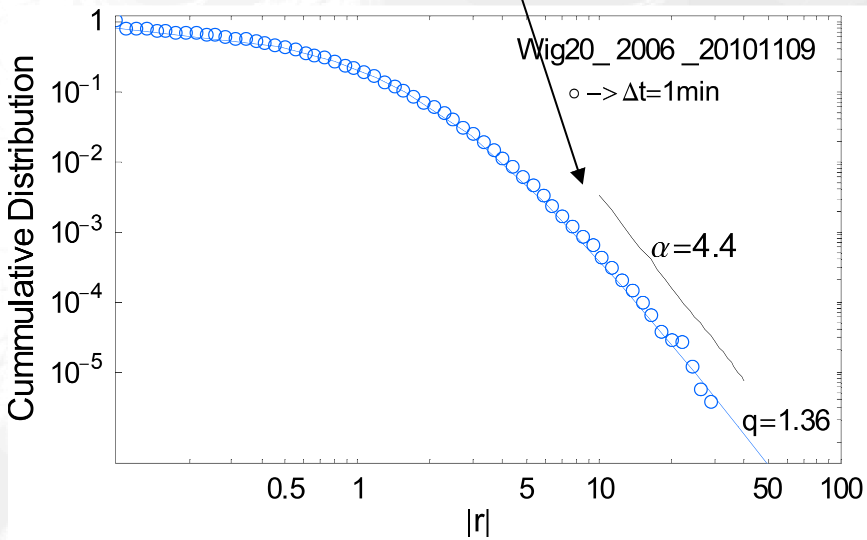
is the Gauss hypergeometric function

q-Gaussian applications

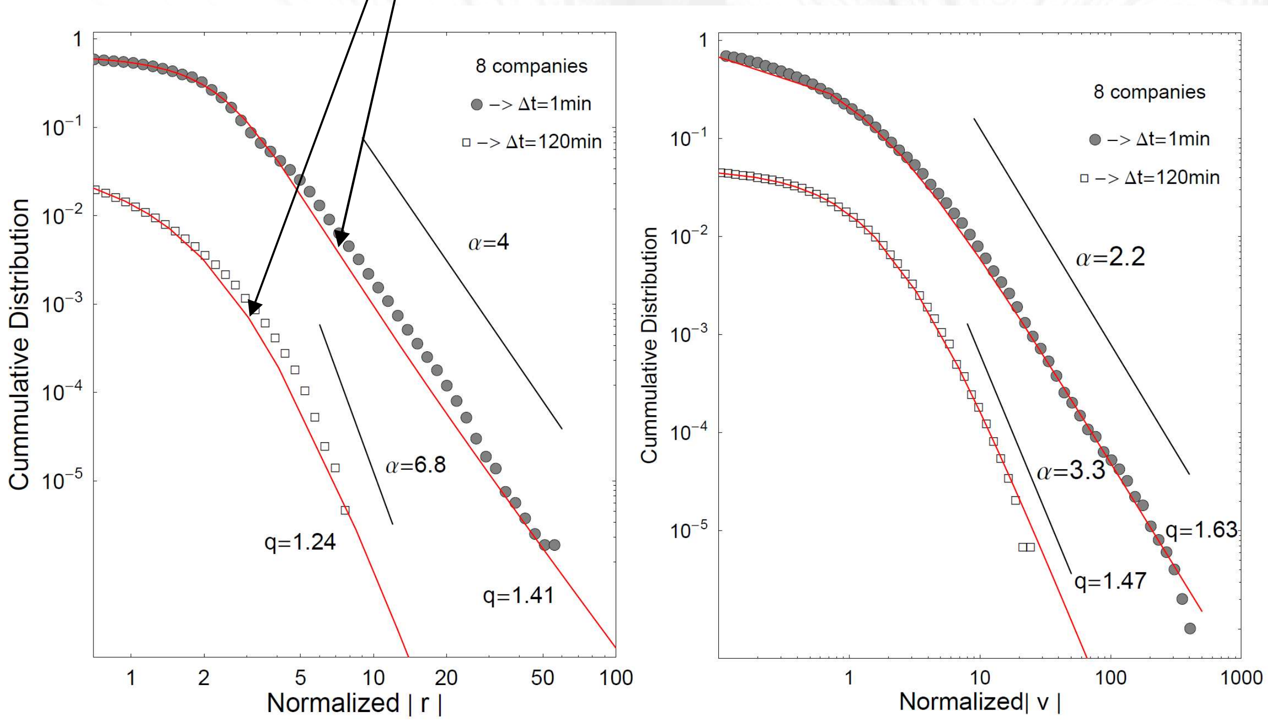


$$P_{\pm}(x) = \mathcal{N}_q \left(\frac{\sqrt{\pi}\,\Gamma\left(\frac{1}{2}(3-q)\,\beta\right)}{2\,\Gamma(\beta)\,\sqrt{\frac{B_q}{\beta}}} \pm (x-\bar{\mu}_q)\,{}_2F_1(\alpha,\beta;\gamma;\delta) \right)$$



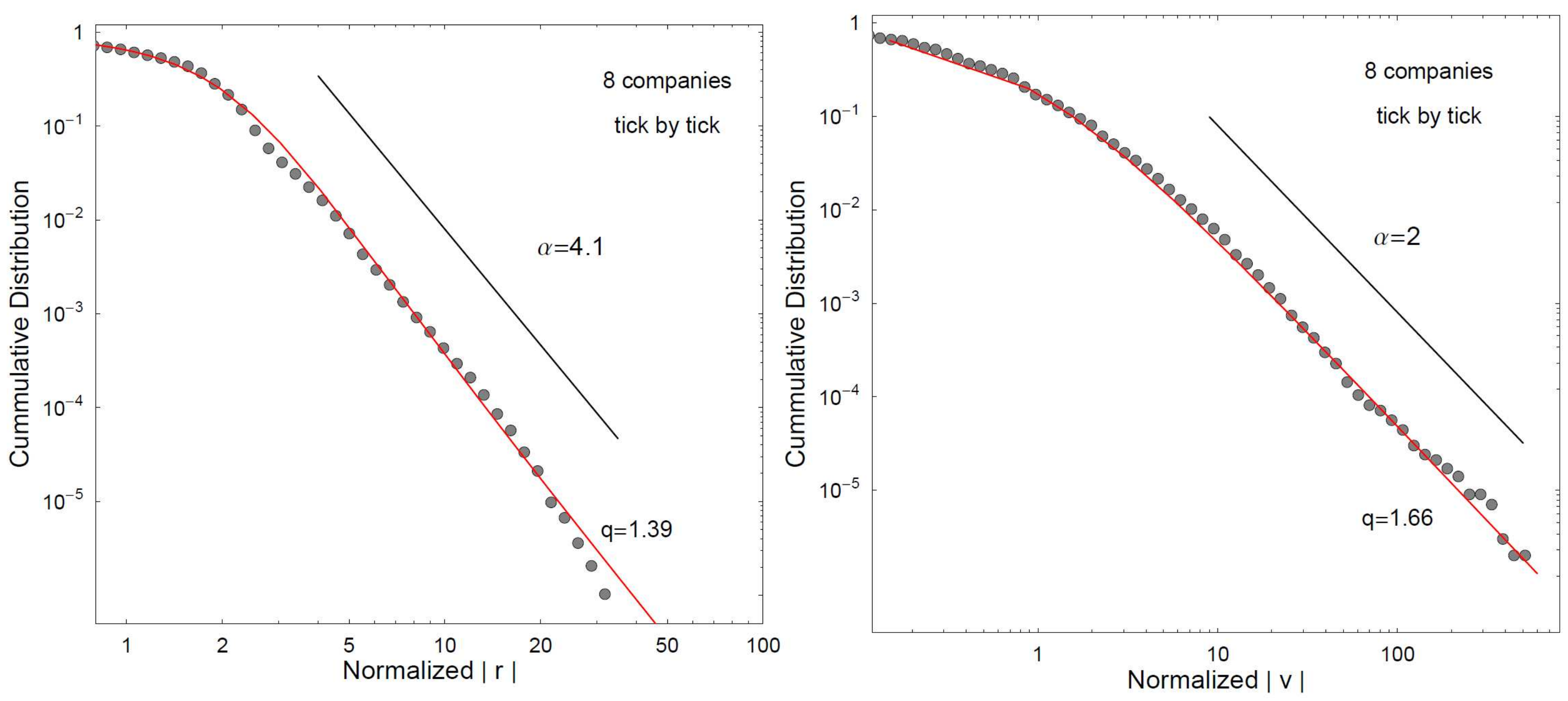


q-Gaussian applications

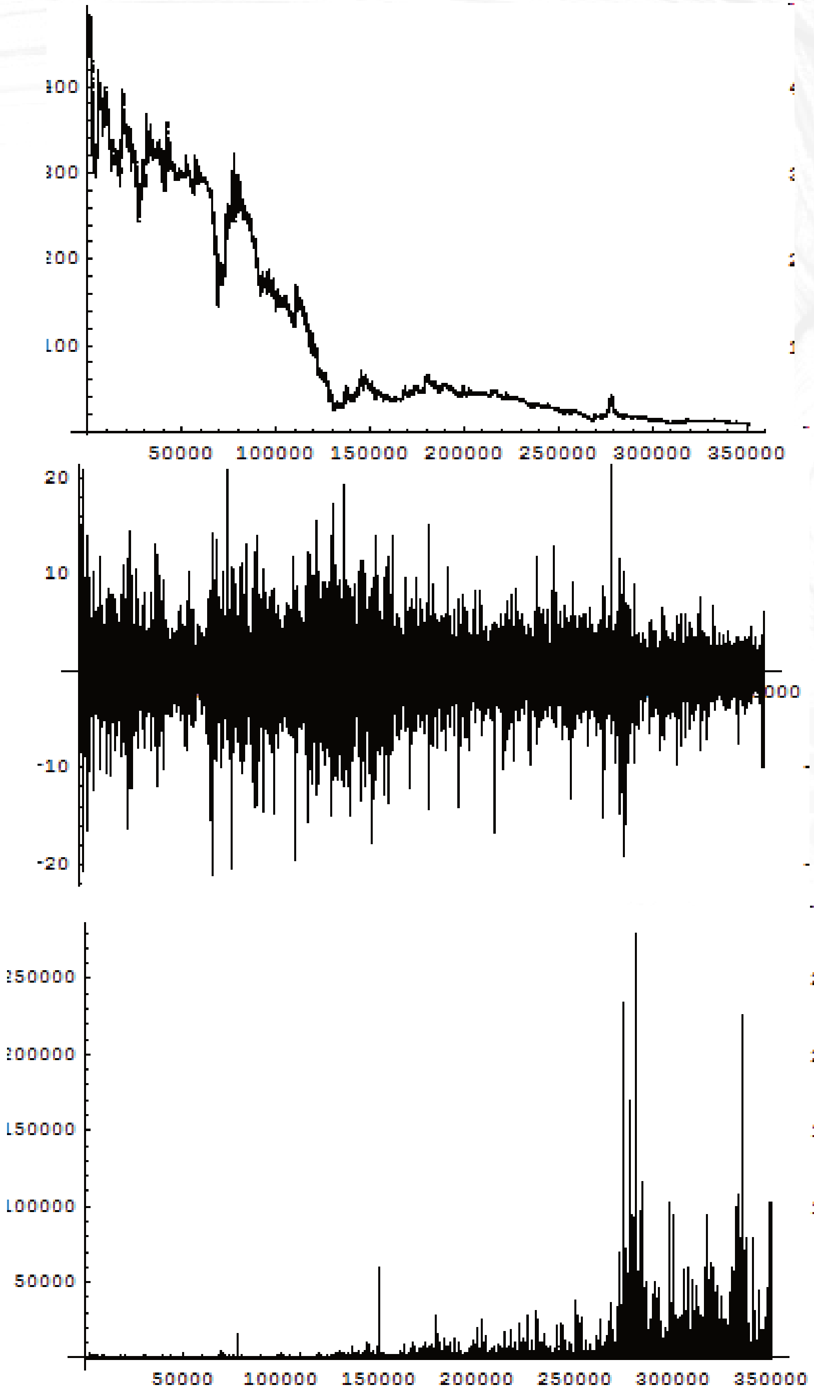


$$P_\pm(x) = \mathcal{N}_q \left(\frac{\sqrt{\pi}\ \Gamma\left(\frac{1}{2}(3-q)\ \beta\right)}{2\ \Gamma(\beta)\ \sqrt{\frac{\mathcal{B}_q}{\beta}}} \pm (x-\bar{\mu}_q)\,{}_2F_1(\alpha,\beta;\gamma;\delta) \right)$$

q-Gaussian applications



$$P_{\pm}(x) = \mathcal{N}_q \left(\frac{\sqrt{\pi}\ \Gamma\left(\frac{1}{2}(3-q)\ \beta\right)}{2\ \Gamma(\beta)\ \sqrt{\frac{\mathcal{B}_q}{\beta}}} \pm (x - \bar{\mu}_q)\ {}_2F_1(\alpha, \beta; \gamma; \delta) \right)$$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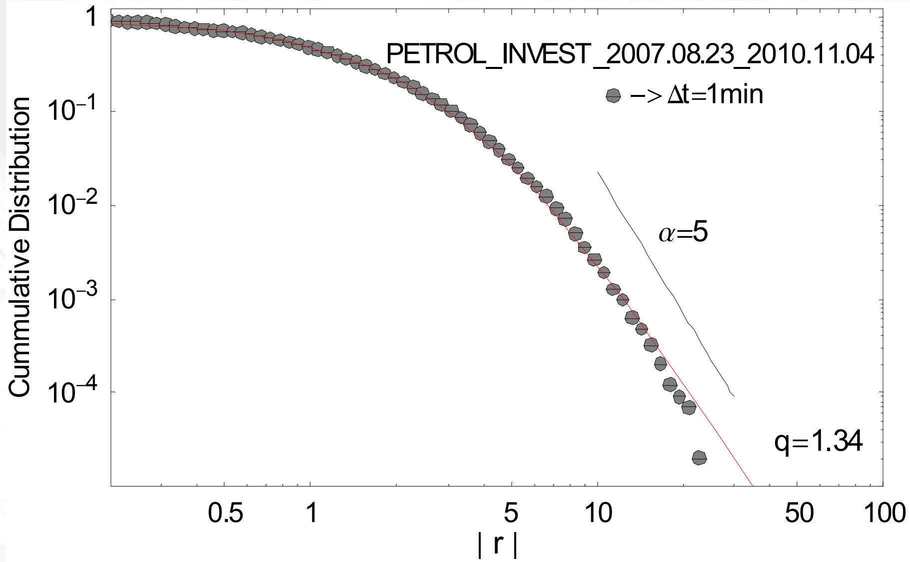
50000 100000 150000 200000 250000 300000 350000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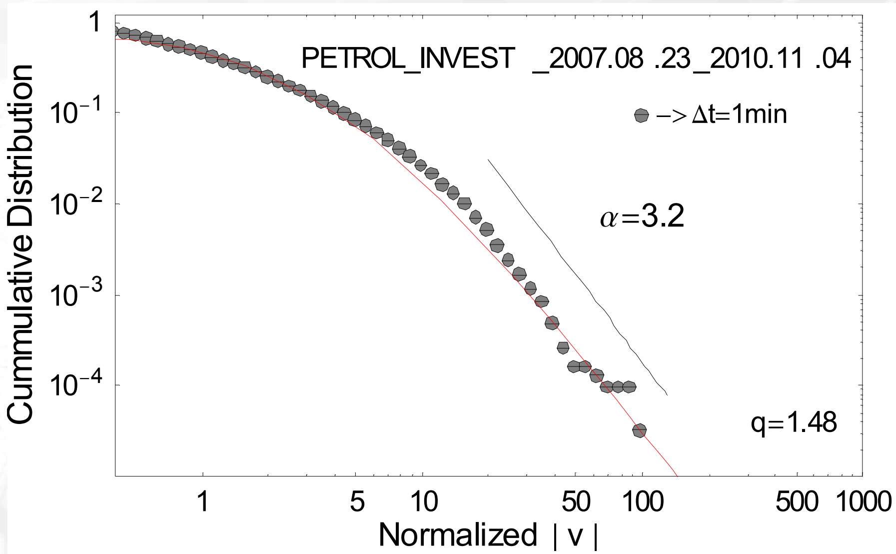
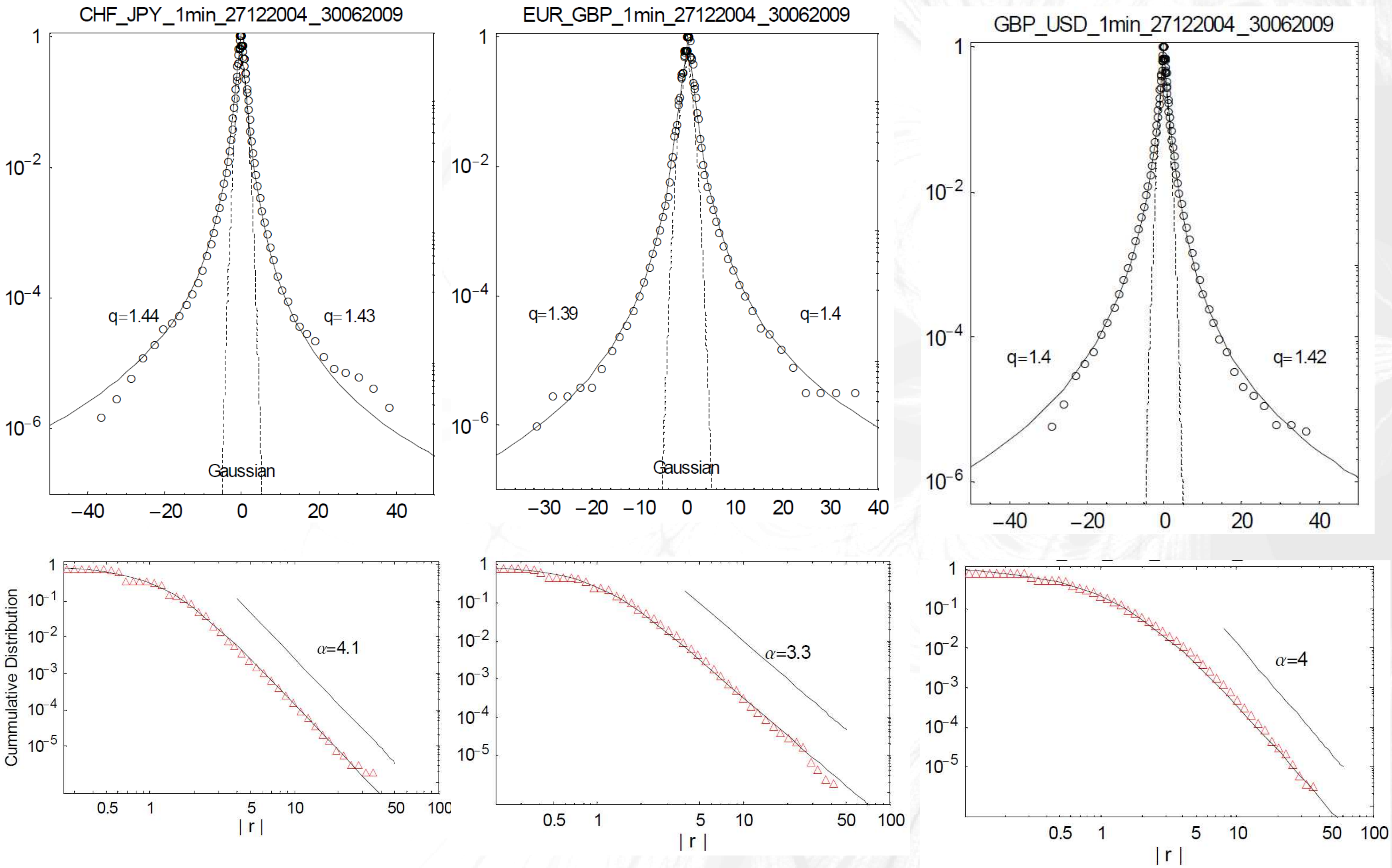
–> Δt=1min
Cummulative Distribution
α=5
q=1.34
| r |


PETROL_INVEST _2007.08 .23_2010.11 .04
–> Δt=1min
Cummulative Distribution
α=3.2
q=1.48
Normalized | v |

Second part

The FX data used in the present analysis are sampled with 1 minute frequency over the period from 2004.01.01- 2009.06.30: CHF/JPY, EUR/GBP, GBP/USD.



Take Home Messages:

$$\frac{\alpha_r}{\alpha_V} \approx 2$$

- This relationship is true even if the inverse cubic power law was not noted

- Not only distributions of returns but also distributions of trading volumes are well describe by q-Gaussians

- Distributions of currency market returns are well modeled by q-Gaussians

Thank you